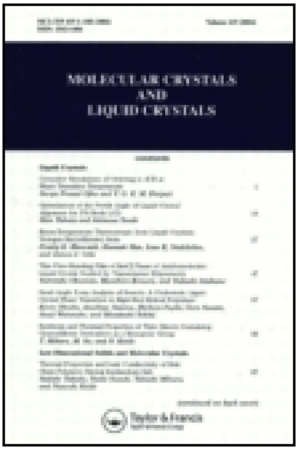

Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

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Version of record first published: 17 Oct 2011.

Mol. Cryst. Liq. Cryst. 1985, Vol. 119, pp. 449-452
0026-8941/85/1194-0449/$10.00/0

AN EXACTLY SOLVABLE ONE-DIMENSIONAL ELECTRON-PHONON SYSTEM

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Abstract An extended Tomonaga-Luttinger model including forward scattering both from electron-electron and electron-phonon interaction is solved exactly using bosonization techniques. We calculate correlation functions for the relevant instabilities and give a phase diagram.

INTRODUCTION : TOMONAGA-LUTTINGER MODEL AND BOSONIZATION METHOD

We consider an extended Tomonaga-Luttinger model including forward scattering (g_2-processes in the "g-ology"language) by electron-electron interaction and electron-phonon interaction. A somewhat simpler model (without electron-electron interaction) was considered previously by Engelsberg and Varga[1] ; the first steps of our calculation are inspired by their treatment.

We immediately write down our Hamiltonian in the boson form

$$H = H_0^{el} + H_0^{ph} + H^{el-el} + H^{el-ph} \tag{1a}$$

$$H_0^{el}=(2\pi v_F/L) \sum_{\nu=\rho,\sigma} \sum_{k>0} \left(\nu_+(k)\nu_+(-k)+\nu_-(k)\nu_-(-k)\right), \tag{1b}$$

$$H_0^{ph}=(1/2) \sum_k (p_k^+ p_k + \omega_k^2 q_k^+ q_k), \tag{1c}$$

$$H^{el-el}=(2/L) \sum_{\nu=\rho,\sigma} \sum_{k>0} g_{2,\nu}(k)\ \nu_+(k)\ \nu_-(-k), \tag{1d}$$

$$H^{el-ph}=L^{-1/2} \sum_k g_{ph}(k)\{\left(\rho_+(k)+\rho_-(k)\right)q_k+\left(\rho_-(-k)+\rho_+(-k)\right)q_k^+\} \tag{1e}$$

The bosonization method is discussed elsewhere[2]; we shall not go into details here. Its physical idea is that the elementary excitations about the ground state are collective particle-hole modes. It can be proved that the density operators obey boson commutation rules and that the eigenstates of the bosonized Hamiltonian form a complete set.

In (1) v_F is the Fermi velocity, the two branches of the linearized dispersion relation (denoted by r = +, -) extend from +∞ to

$-\infty$. It is essentially the linearized dispersion relation together with the limitation to small momentum transfer scattering that allows an exact solution of the problem. $\nu_r(k)=\rho_r(k)$, $\sigma_r(k)$ stands for charge or spin density excitations which obey the commutation rule

$$\left(\nu_r(k),\ \nu'_{r'}(k')\right) = -\delta_{\nu,\nu'}\ \delta_{r,r'}\ \delta_{k,k'}(rkL/2\pi) \qquad (2)$$

$g_{2,\nu}(k)$ is the charge or spin density part of the electronic coupling constant and $g_{ph}(k)$ is the electron-phonon coupling. (The phonons do not couple to the spin density). p_k^+ is the canonically conjugate momentum to the phonon coordinate q_k.

DIAGONALIZATION OF THE HAMILTONIAN

Following Engelsberg and Varga[1] we perform a canonical transformation of the density operators to "phonon-like" coordinates ($\Omega_k = v_F\ |k|$)

$$\nu_r(k) = (L|k|\Omega_k/4\pi)^{1/2}\left(Q^+_{k,\nu}-(ir/\Omega_k)\mathrm{sign}(k)P^+_{k,\nu}\right) \qquad (3)$$

Solving now the Heisenberg equation of motion we obtain the following eigenvalues (cf fig. 1)

$$\lambda_\rho^2(k) = C^2_{k,\sigma}\ \Omega_k^2 \qquad (4a)$$

for the spin density excitations,

$$\lambda^2(k)=\{C^2_{k,\rho}\Omega_k^2+\omega_k^2+\left((C^2_{k,\rho}\Omega_k^2-\omega_k^2)^2+4A_k\ B_k\right)^{1/2}\}/2 \qquad (4b)$$

for the "charge density-like" excitations, and

$$\lambda^2_{ph}(k)=\{C^2_{k,\rho}\Omega_k^2+\omega_k^2-\left((C^2_{k,\rho}\Omega_k^2-\omega_k^2)^2+4A_kB_k\right)^{1/2}\}/2 \qquad (4c)$$

for the "phonon-like" excitations of the interacting system.

$$B_k=\left(1-g_{2,\rho}(k)/\pi v_F\right)A_k=\left(1-g_{2,\rho}(k)/\pi v_F\right)\left(2|k|\Omega_k/\pi\right)^{1/2}g_{ph}(k), \qquad (5)$$

$$C^2_{k,\nu} = 1 - \left(g_{2,\nu}(k)/\pi v_F\right)^2, \qquad (6)$$

and the $Q_{k,\nu}$, $P_{k,\nu}$ are renormalized to

$$\tilde{Q}_{k,\nu} = (Z_{k,\nu}\Omega_k)^{1/2}\ Q_{k,\nu}\ , \qquad (7)$$

$$\tilde{P}_{k,\nu} = (Z_{k,\nu}\ \Omega_k)^{-1/2}\ P_{k,\nu}, \qquad (8)$$

$$Z_{k,\nu} = \left(1 - g_{2,\nu}(k)/\pi v_F\right)/\lambda_\nu\ . \qquad (9)$$

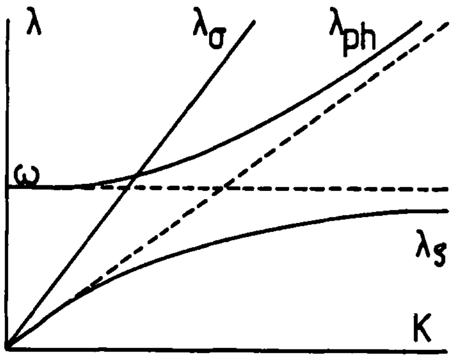

FIGURE 1 : Eigenvalues of the Hamiltonian (1) for molecular phonons. The dashed lines are for $g_{ph} = 0$.

INSTABILITIES IN THE SYSTEM

A strictly one-dimensional system cannot undergo a transition to a long range ordered phase. We therefore look at the divergences of the correlation functions of CDW-, SDW-, SS-, and TS-fluctuations

$$R_j(xt,00) = - i\langle T\ \{O_j(xt)\ O_j^+(00)\}\rangle \tag{10}$$

The operator $O^+_{CDW}(xt)$ creates a CDW-fluctuation at (x,t), etc...[3].

Looking only at the asymptotic behaviour $(x\to\infty)$ of these correlation functions and setting t = 0 we obtain a power law

$$R_j(x0,00) \sim \exp(2ik_F x)|x|^{-2+\alpha_j} \quad , j = CDW, SDW \ . \tag{11a}$$

$$R_j(x0,00) \sim |x|^{-2+\alpha_j} \quad , j = SS, TS \ ; \tag{11b}$$

or in k-space, for small k,

$$R_j(k,\omega) \sim \{\max(v_F k, \omega)\}^{-\alpha_j} \ . \tag{12}$$

For the density wave response functions k is taken relative to $2k_F$. α_j is given by

$$\alpha_{CDW}=2-\{(1-g_{2,\sigma}/\pi v_F)/(1+g_{2,\sigma}/\pi v_F)\}^{1/2} -(1-g_{2,\rho}/\pi v_F)/\lambda'_\rho \ , \tag{13a}$$

$$\alpha_{SDW}=2-\{(1+g_{2,\sigma}/\pi v_F)/(1-g_{2,\sigma}/\pi v_F)\}^{1/2}-(1-g_{2,\rho}/\pi v_F)\lambda'_\rho \ , \tag{13b}$$

$$\alpha_{SS} =2-\{(1-g_{2,\sigma}/\pi v_F)/(1+g_{2,\sigma}/\pi v_F)\}^{1/2}-\lambda'_\rho/(1-g_{2,\rho}/\pi v_F), \tag{13c}$$

$$\alpha_{TS} =2-\{(1+g_{2,\sigma}/\pi v_F)/(1-g_{2,\sigma}/\pi v_F)\}^{1/2}-\lambda'_\rho/(1-g_{2,\rho}/\pi v_F). \tag{13d}$$

$\lambda'_\rho = \lim_{k\to 0} \lambda_\rho(k)/\Omega_k$ and all the coupling constants are taken at k = 0.

For $k\to 0$ we can get simple expressions for λ'_ρ : if we consider the forward scattering part of molecular (dispersionless) phonon modes, we obtain

$$\lambda'_\rho \approx \{1-(g_{2,\rho}/\pi v_F)^2-(2g_{ph}^2/\pi v_F\omega_0^2)(1-g_{2,\rho}/\pi v_F)\}^{1/2} \quad , \qquad (14)$$

and for acoustical phonons (with sound velocity v_s)

$$\lambda' \approx \left(1-(g_{2,\rho}/\pi v_F)^2+(v_s/v_F)^2(2g_{ph}^2/\pi v_F)(1+g_{2,\rho}/\pi v_F)^{-1}\right)^{1/2} \; . \qquad (15)$$

The phase diagram for molecular phonons is given in fig. 2.

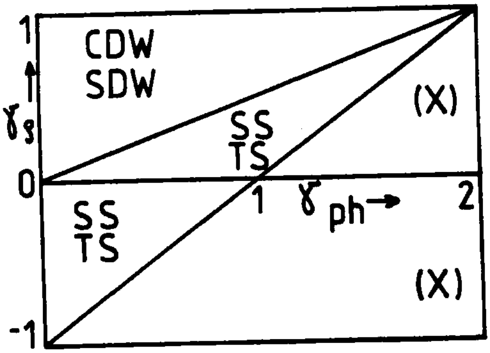

FIGURE 2 : Divergent fluctuations in the $g_{2,\rho}$ -g_{ph} plane for spin isotropic electron-electron interaction ($g_{2,\sigma}$=0). $\gamma_\rho = g_{2,\rho}/\pi v_F$ and $\gamma_{ph} = 2g_{ph}^2/\pi v_F\ \omega^2$. In the region (X) the model is not well defined.

Note that (i) in the exponents of the correlation functions the phonon coupling terms add to the square of the electronic couplings ; (ii) acoustical phonons give contributions only in second order in v_s/v_F ; (iii) for spin anisotropic electron-electron interaction singlett instabilities are favoured for $g_{2,\sigma} > 0$, triplett instabilities for $g_{2,\sigma} < 0$.

This work profited from financial support by Studienstiftung des Deutschen Volkes (Bonn) and Ministry of Industry and Research (Paris).

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